Learning Objectives

1. **Explain** meaning of element-by-element operations
2. **Identify** situations where the standard operators in MATLAB (when used with arrays) are reserved for linear algebra, which is not always element-by-element
3. **Apply** dot operators for
   a. The six cases where linear algebra is not element-by-element and therefore dot operators are needed to produce element-by-element calculations

Topics

**Read Chapter 3.4 – 3.8 of the MATLAB book before coming to class.** This preparation material is provided to supplement this reading.

Students will learn a basic understanding of array operations in MATLAB. This material contains the following:

1. **Matrix Arithmetic Overview**
   a. Matrix Math (beyond this class)
   b. Element-by-Element Arithmetic
2. Scalar-Array Arithmetic
3. Array-Array Arithmetic
4. Dot Operators
5. Element-by-Element Examples
6. Built-In Array Functions
7. Application of Array Operations

1. **Matrix Arithmetic Overview**

There are two types of matrix arithmetic:

1. Matrix math is used in Linear Algebra to solve simultaneous equations. This is beyond the scope of this course, but can be read about in Chapter. 3.2 and 3.3.

2. This course introduces element-by-element arithmetic.

   • If $A = [a \ b \ c]$ and $B = [x \ y \ z]$, then $A + B = [a+x \ b+y \ c+z]$
If A and B are matrices and s is a scalar, then:

- There are only six cases where matrix math and element-by-element arithmetic differ:
  - $A \times B$  $A / B$  $A \wedge B$  $A \wedge s$  $s \wedge A$  $s / A$

- For these cases, the standard operator has been chosen to represent matrix arithmetic, so we need a new symbol for element-by-element arithmetic (dot operators):
  - $A \cdot B$  $A \cdot / B$  $A \cdot \wedge B$  $A \cdot \wedge s$  $s \cdot \wedge A$  $s \cdot / A$

The arrays **must** be the same size for matrix arithmetic.

**Element-by-element operations for row vectors:**

If: $a = [a_1 \ a_2 \ a_3]$ and $b = [b_1 \ b_2 \ b_3]$

Then:

- $a \cdot \times b = [a_1 \cdot \times b_1 \ a_2 \cdot \times b_2 \ a_3 \cdot \times b_3]$
- $a \cdot / b = [a_1 / b_1 \ a_2 / b_2 \ a_3 / b_3]$
- $a \cdot \wedge b = [a_1 \wedge b_1 \ a_2 \wedge b_2 \ a_3 \wedge b_3]$
- $a \cdot \wedge 2 = [a_1 \wedge 2 \ a_2 \wedge 2 \ a_3 \wedge 2]$
- $2 / a = [2 / a_1 \ 2 / a_2 \ 2 / a_3]$

**2. Dot Operators**

Given the matrices $A$ and $B$:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

Then, the following dot operations are true:

**Multiplication**

$$A \cdot \times B = \begin{bmatrix} A_{11}B_{11} & A_{12}B_{12} & A_{13}B_{13} \\ A_{21}B_{21} & A_{22}B_{22} & A_{23}B_{23} \\ A_{31}B_{31} & A_{32}B_{32} & A_{33}B_{33} \end{bmatrix}$$

**Division**

$$A \cdot / B = \begin{bmatrix} A_{11}/B_{11} & A_{12}/B_{12} & A_{13}/B_{13} \\ A_{21}/B_{21} & A_{22}/B_{22} & A_{23}/B_{23} \\ A_{31}/B_{31} & A_{32}/B_{32} & A_{33}/B_{33} \end{bmatrix}$$

(vector by vector)
Division

\[6 ./ A = \begin{bmatrix}
\frac{6}{A_{11}} & \frac{6}{A_{12}} & \frac{6}{A_{13}} \\
\frac{6}{A_{21}} & \frac{6}{A_{22}} & \frac{6}{A_{23}} \\
\frac{6}{A_{31}} & \frac{6}{A_{32}} & \frac{6}{A_{33}}
\end{bmatrix}\]

(scalar by vector)

Exponential

\[A \cdot ^2 = \begin{bmatrix}
(A_{11})^2 & (A_{12})^2 & (A_{13})^2 \\
(A_{21})^2 & (A_{22})^2 & (A_{23})^2 \\
(A_{31})^2 & (A_{32})^2 & (A_{33})^2
\end{bmatrix}\]

(vector to scalar)

If you forget a dot when one is required, you will either get a cryptic error message (involving matrix dimensions) or you will get the wrong answer.

For certain operations, matrix math is the same as element-by-element arithmetic, so dot operators are optional. This is true for the following operations:

- s+A
- s-A
- A-s
- A+B
- A-B
- s*A
- A/s

3. Scalar-Array Arithmetic

**Scalar-Array Addition and Subtraction**

The scalar operates on each element in the array.

For example, given the following matrix \(M\) and the scalar \(s = 2\) we would get the following results for addition and subtraction.
Scalar-Array Multiplication and Division

The scalar operates on each element in the array.

For example, given the same matrix $M$ and scalar $s = 2$ we would get the following results for multiplication and division.

$$
\begin{align*}
\text{>> } M &= \\
M &= \\
1 & 2 \; 3 \\
4 & 5 \; 6 \\
7 & 8 \; 9 \\
\text{>> } a &= s + M \\
a &= \\
3 & 4 \; 5 \\
6 & 7 \; 8 \\
9 & 10 \; 11 \\
\text{>> } b &= s - M \\
b &= \\
1 & 0 \; -1 \\
-2 & -3 \; -4 \\
-5 & -6 \; -7 \\
\text{>> } c &= M - s \\
c &= \\
-1 & 0 \; 1 \\
2 & 3 \; 4 \\
5 & 6 \; 7 \\
\text{>> } d &= s \times M \\
d &= \\
2 & 4 \; 6 \\
8 & 10 \; 12 \\
14 & 16 \; 18 \\
\text{>> } d &= M \div s \\
d &= \\
0.5000 & 1.0000 \; 1.5000 \\
2.0000 & 2.5000 \; 3.0000 \\
3.5000 & 4.0000 \; 4.5000
\end{align*}
$$
4. Array-Array Arithmetic

Array-Array Addition and Subtraction

The arrays being used in the operation must have the same size.

The sum (difference) of two arrays is obtained by adding (subtracting) corresponding array elements, resulting in a new array of the same size.

$$\begin{array}{c}
\text{>> } A = [1 \ 2; \ 3 \ 4] \\
A = \\
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
\end{bmatrix}
\end{array}$$

$$\begin{array}{c}
\text{>> } B = [1 \ -1; \ 2 \ 0] \\
B = \\
\begin{bmatrix}
1 & -1 \\
2 & 0 \\
\end{bmatrix}
\end{array}$$

$$\begin{array}{c}
\text{>> } C = A + B \\
C = \\
\begin{bmatrix}
2 & 1 \\
5 & 4 \\
\end{bmatrix}
\end{array}$$

$$\begin{array}{c}
\text{>> } A - B \\
\text{ans} = \\
\begin{bmatrix}
0 & 3 \\
1 & 4 \\
\end{bmatrix}
\end{array}$$
5. Element-by-Element Examples

Given the following matrices A and B:

\[
A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -2 & 5 \end{bmatrix}
\]

...then the following operations are true:

\[
A \times B = \begin{bmatrix} 1 & 4 & -3 \\ 8 & -10 & 30 \end{bmatrix}, \quad A / B = \begin{bmatrix} 1.0000 & 1.0000 & -3.0000 \\ 2.0000 & -2.5000 & 1.2000 \end{bmatrix}, \quad B^3 = \begin{bmatrix} 1 & 8 & -1 \\ 8 & -8 & 125 \end{bmatrix}
\]
6. Built-In Array Functions
MATLAB has many built-in functions that can be used with arrays. Some of them are (A is a vector):

- \text{max}(A) \quad \text{Returns the largest element in } A
- \text{min}(A) \quad \text{Returns the smallest element in } A
- \text{mean}(A) \quad \text{Returns the average value of the elements in } A
- \text{sum}(A) \quad \text{Returns the sum of the elements of } A
- \text{length}(A) \quad \text{Returns the number of elements in } A
- \text{sort}(A) \quad \text{Sorts the elements of } A

The \textbf{Help} window gives information about many other built-in MATLAB functions.

EXAMPLES OF BUILT IN FUNCTIONS:

\begin{verbatim}
>> A = [8 2 9 5 14 10]
A =
 8     2     9     5    14
 10
\end{verbatim}
>> max(A)
an =
 14

>> min(A)
an =
  2

>> mean(A)
an =
   8

>> sum(A)
an =
  48

>> length(A)
an =
   6

>> sort(A)
an =
  2  5  8  9 10 14
>> mass = [2 4 5 10 20 50];
>> force = [12.5 23.2 30 61 116 294];
>> mu = force ./ (9.81 * mass)
mu =
   0.6371   0.5912   0.6116   0.6218   0.5912   0.5994
>> mu_ave = mean(mu)
meu_ave =
   0.6087